
Mathematical model of multi-frequency piezoresonance oscillation system

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Abstract: The paper represents the basic model of multi-frequency piezoresonance oscillation system (MPOS) – the piezoresonance devices (PRD) core, which enables to study the processes of establishing multi-frequency oscillation mode and its stability. The basic structure of multi-channel multi-frequency PRD core, which is based on principles of filter schemes, is proposed, and the main designations are entered. The peculiarities of truncated differential equations for amplitude, phase and auto-bias voltage of MPOS for the quantity of simultaneously generated frequencies $m \geq 2$ are examined. On the example of three-frequency mode of oscillation under polynomial approximation of transferable characteristics of active elements the characteristic cases of establishing oscillations in MPOS are represented. The area of a steady three-frequency oscillating behavior is defined and the assessment of time of establishment of oscillations and value of group runout of frequencies is made. Received results enable to form a new approach to construction of piezoresonance devices with controlled dynamics, which are represented in the form of adaptive controlled systems with predictive standard model and develop on its basis the new class of invariant to destabilizing disturbing PRD factors. On the basis of such approach there is the principle of using natural redundancy in multi-frequency basis of PRD core – multi-frequency oscillation system, which enables not only to synthesize the system with current identification of disturbing factors on basis of instruments of invariance theory, but also do the adaptation of PRD in accordance with their influences.

Keywords: Stabilization of Oscillations, Multi-Frequency Quartz Oscillatory System, Invariant Piezoresonance System

1. Introduction

Anisotropic properties of crystalline piezoelements cause different types of elastic ties of mechanical stresses and deformations, which lead to appearance of unwanted resonances in piezoresonance devices (PRD). Usually multi-frequency of crystalline piezoelements, taking into account destabilizing factors (DF), which have an effect on PRD, is considered as undesirable effect, which is tried to be removed by means of structural and technological methods, which is often at variance with demands of mass production.

On the basis of conducted research [1-4] a new approach has been formed to provide the invariance of piezoresonance devices. Such approach is based on the conception of PRD as a dynamic object with natural redundancy in the frequency basis (multi-frequency) and current identification of reversible on PRD the destabilizing factors. At the same time, quartz resonator (QR) has frequency –

determining (stabilizing) function and the additional measuring function, which enables to accomplish continuous identification of DF.

Magnitude estimation of destabilizing disturbing factors are determined by special algorithmic processing of multi-frequency signals, which defined such approach as multi-frequency – algorithmic [2,4]. These valuations of DF can be used for solving the tasks of compensation, statting or as a result of solving individual task of synchronous measuring of several physical quantities in local volume of measurement field [8]. The theoretic basis of multi-frequency – algorithmic approach is the fundamental works on the theory of invariance and its use in the field of measuring technique and automatic devices [3]. The material basis of multi-frequency – algorithmic approach is the rapid development of microelectronic and microprocessor technique, the use of which in the complex with multi-frequency excitation of QR, is the “revolutionary tendency” in precision piezoelectronics [2].

Recently the development of multi-frequency – algorithmic methods of provision the invariance of PRD led to their use in several two- frequency (two- mode) piezoresonance oscillators and measurement systems with current identification of temperature influence. However, the research in such field [5-15] do not take into account the whole spectrum of specific conditions of modern PRD functioning, which reveals in sharp changes in temperature rate and substantial vibration and mechanical effects, which makes the task of providing the invariance in the conditions of parametrical non-stationarity essentially difficult. At that the following mechanisms, which define the dynamics of multi-frequency PRD, can be defined [16,17]:

1. The conversion to multi-frequency oscillation mode (MOM) of QR caused by competition of oscillations in the PRD excitation channels reduces the oscillation stability and complicates considerably the character of transient processes, which leads to extending the time for setting MOM as compared to one- frequency mode [18];

2. The use of MOM under the conditions of temperature DF requires firmer controlling of summary power of excitation and the thermal QR condition. The presence of rapidly changing thermal flows, the considerable temperature gradients leads to irregular heating of QR piezoelement and causes specific thermo dynamic shift of QR frequency, which distorts sharply its temperature and frequency characteristics, which altogether reduces the long-term stability of PRD oscillations substantially [19,20];

3. Though quartz resonator is inert to temperature DF, the inertial properties are negligible towards mechanical forces (such as impacts, vibration, acoustic noises). The effect of vibration and mechanical DF causes parasite frequency modulation of PRD oscillation (the decrease of short-term stability), which especially develops in frequency measuring transducers where QR with enchanted force sensitivity is used [21].

Listed above factors cause the necessity of conducting further research on expanding functional potential of multi-frequency algorithmic methods, which are based on introducing PRD as dynamic object. This requires the development of generic model of multi-frequency PRD for studying the dynamics on the stage of oscillation determination as well as under the conditions of temperature and vibration influence for multi-frequency oscillation mode with the quantity of generated frequency $m \geq 2$.

Present work introduces the basic model of multi-frequency piezoresonance oscillatory system (MPOS) – PRD core, which allows to analyze the processes of establishing multi-frequency oscillation mode and its stability. The second section proposes the basic structure of multi-channel multi-frequency PRD core and introduces new indications. In the third section features of an outputs of the truncated differential equations are considered for amplitudes, phases and auto-bias voltages of MPOS for the quantity of simultaneously generated frequency $m \geq 2$.

The fourth section represents characteristic cases of establishing oscillations in MPOS on the example of

three-frequency oscillation mode under polynomial approximation of transferable characteristics of active elements. The field of stable three-frequency oscillation mode is defined, and the estimation of the time of establishing the oscillations and value of group frequency overshoot is done. Further the discussion of the most considerable received results is given.

2. Multi-frequency Core Structure of a Piezoresonance Oscillation System

High effectiveness of multi-frequency generating mode is provided by condition of high stability in generated oscillations. This is possible only with using filtering schemes in which QR is embedded in feedback circuit and is excited in the frequencies close to those of its own consequent resonances. At limited quantity of generated oscillations the other oscillator schemes can be used, for example those of oscillating type [22].

The basic core architecture represents MPOS to have principles of creating filtering schemes implemented (fig. 1). It incorporates passive multi-frequency quartz quadripole unit (MQU) on the base of quartz resonator with m - frequencies generating Z_{qj} and n excitation channels embedding the generalized non-linear component (NLC) and phasing selective feedback circuit (FBC). The automatic bias circuits with complex equivalent resistance Z_{b_j} are used for stabilizing NLCi operational mode. The selective non-linear circuits FBC with gain $K_{ji}(j\omega, u_\Sigma, \tau)$, $j = \overline{1, m}$, $i = \overline{1, n}$, except for their function to set required amplitude-phase ratio in excitation channels, provide significant reducing competition in oscillations due to their own selective properties $K_{ji}(j\omega)$ and also automatic adjustment $K_{ji}(u_\Sigma)$ of oscillation amplitudes for fixing the specified (ultimately acceptable) power dissipation on QR.

Figure 1 has the following symbols' identifications:

$z_{in_\Sigma} = R_{in_\Sigma} / (1 + j\omega\tau_{in_\Sigma})$, $R_{in_\Sigma}^{-1} = \sum_i R_{in_i}^{-1}$, $C_{in_\Sigma} = \sum_i C_{in_i}$ – the complex equivalent total resistance of partial FC input circuits;

$$z_{out_\Sigma} = R_{out_\Sigma} / (1 + j\omega\tau_{out_\Sigma}), \quad R_{out_\Sigma}^{-1} = \sum_i R_{out_i}^{-1},$$

$C_{out_\Sigma} = \sum_i C_{out_i}$ – the complex equivalent total resistance of output NLCi circuits;

$z_{ab_j} = R_{ab_j} / (1 + j\omega\tau_{ab_j})$ – complex resistance of auto-bias circuit i ;

$\tau_{ab_j} = R_{ab_j} C_{ab_j}$, $\tau_{in_\Sigma} = R_{in_\Sigma} C_{in_\Sigma}$, $\tau_{out_\Sigma} = R_{out_\Sigma} C_{out_\Sigma}$ – time constants;

$$i_\Sigma = \sum_i i_{out_i}(e_i) \text{ – total current of NLCi;}$$

$e_i(\tau) = \sum_j v_i(\tau) + E_i + \tilde{E}_i(\tau)$ – control voltage in NLC_i input,

where E_i , $\tilde{E}_i(\tau)$ – constant and variable components of auto-bias voltage;

$u_\Sigma(\tau) = U_0(\tau) + \sum_j U_j(\tau) \cdot \cos[\omega_j \tau + \varphi_j(\tau)]$ – the total voltage in FC circuit input, where $U_j(\tau)$, ω_j and $\varphi_j(\tau)$ – envelope, frequency and phase of oscillation j correspondently, $\tau = t - t_0$ – time interval from initial moment t_0 (MPOS startup moment) [16].

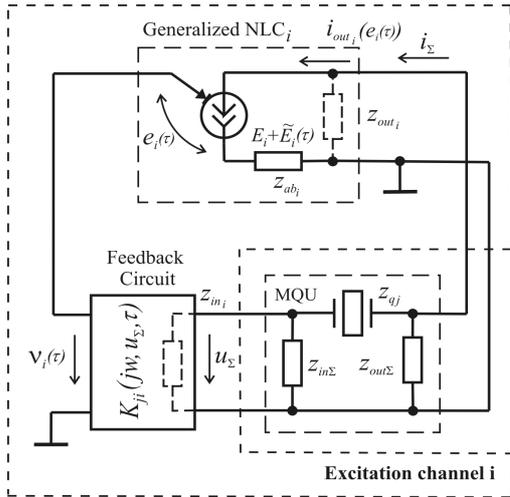


Figure 1. The basic core architecture.

3. Truncated Differential Equations for Amplitudes and Phases of Oscillations

MPOS operation (fig. 1) is described by the system of differential equations:

$$u_\Sigma = Z(p) \cdot \sum_i i_{out_i} \left(\sum_j K_{ji} \cdot u_\Sigma, \tilde{E}_i, E_i \right) \quad (1)$$

$$E_i = -z_{ab_i}(p) \cdot i_{out_i} \left(\sum_j K_{ji} \cdot u_\Sigma, \tilde{E}_i, E_i \right) \quad (2)$$

$$Z(p) = (z_{in_\Sigma}(p) z_{out_\Sigma}(p)) / (z_{in_\Sigma}(p) + z_{out_\Sigma}(p) + z_q(p)) \quad (3)$$

– symbolic gain of MQU, representing control resistance of MPOS; $p \equiv d/dt$.

Representing control resistance $Z(p)$ (3) as the ratio

$$Z(p) = \frac{\delta \cdot P(p, \delta)}{Q(p, \delta)}, \quad (4)$$

where $P(p, \delta)$, $Q(p, \delta)$ – polynomials of p ; δ – low parameter, physically determined for resonance systems as damping ratio, and taking into account that in the general case solving system (1) is as sum of oscillations with

frequencies close to resonance ones of QR ω_{q_j}

$$u_\Sigma = \sum_{j=1}^m U_j(\tau) \cos(\omega_{q_j} \tau + \varphi_j(\tau)), \quad (5)$$

where $U_j(\tau)$ and $\varphi_j(\tau)$ – slowly changing parameters, the output current of NLC_i with low parameter δ accuracy can be written as:

$$i_{out_i}(e_i) = I_{oi}(V_{ji}, E_i, \tilde{E}_i) + \sum_{j=1}^m I_{ji}(V_{ji}, E_i, \tilde{E}_i) \cos[\omega_{q_j} \tau + \psi_{ji}(\tau)], \quad (6)$$

where I_{oi} – constant component; $I_{ji} = \bar{S}_{ji}(V_{ji}, E_i, \tilde{E}_i) \cdot V_{ji} = \bar{S}_{ji}(V_{ji}, E_i, \tilde{E}_i) \cdot U_j \cdot K_{ji}$; $\psi_{ji} = \varphi_j + \Delta\varphi_{ji}$;

K_{ji} , $\Delta\varphi_{ji}$ – gain and phase shift of circuit i of FBC for resonance frequency j of QR;

$\bar{S}_{ji}(V_{ji}, E_i, \tilde{E}_i) = \frac{1}{V_{ji}} L_j [i_{out_i}(V_{ji}, E_i, \tilde{E}_i)]$ – mean slope for oscillation j ; $L_j[\bullet]$ – mean operator.

Having entered complex amplitudes $\dot{U}_j = U_j \exp(j\varphi_j)$, $\dot{I}_{ji} = I_{ji} \exp(j\psi_{ji})$ and complex FBC gain $\dot{K}_{ji} = K_{ji} \exp(j\Delta\varphi_{ji})$, and split real and imaginary units the truncated equations for amplitudes, MPOS oscillation phases and auto-bias voltages become [23,24]:

$$T_j \frac{dU_j}{dt} = \left[R_{e_j} K_{\phi_j} \sum_{i=1}^n \bar{S}_{ji}(V_{ji}, \tilde{E}_i, E_i) \times \right. \\ \left. \times K_{ji} \cos(\Delta\varphi_{ji} + \Delta\varphi_j) - 1 \right] \cdot U_j; \quad (7)$$

$$T_j \frac{d\varphi_j}{dt} = R_{e_j} K_{\phi_j} \sum_{i=1}^n \bar{S}_{ji}(V_{ji}, \tilde{E}_i, E_i) \times \\ \times K_{ji} \sin(\Delta\varphi_{ji} + \Delta\varphi_j) - \Delta\omega_j T_j \quad (8)$$

$$T_{ab_i} \frac{d\tilde{E}_i}{dt} = -[R_{ab_i} I_{oi}(V_{ji}, \tilde{E}_i, E_i) + \tilde{E}_i]; \quad j = \overline{1, m}; i = \overline{1, n}, \quad (9)$$

or as generalized matrix

$$\mathbf{T} \frac{d\mathbf{U}}{dt} = [\mathbf{G}_R \cdot \mathbf{R} - \mathbf{E}_{(mm)}] \cdot \mathbf{U}; \quad (10)$$

$$\mathbf{T} \frac{d\mathbf{\Phi}}{dt} = [\mathbf{G}_I \cdot \mathbf{R} - \Delta] \cdot \mathbf{E}_{(m1)}; \quad (11)$$

$$\mathbf{T}_{ab} \frac{d\tilde{\mathbf{E}}}{dt} = -(\mathbf{R}_{ab} \cdot \mathbf{I}_0 + \tilde{\mathbf{E}}), \quad (12)$$

where $\mathbf{T} = \text{diag}(T_1, \dots, T_m)$, $\mathbf{T}_{ab} = \text{diag}(\tau_{ab_1}, \dots, \tau_{ab_n})$ – time constant matrixes of partial oscillation system $T_j = 2/\omega_{q_j} \delta_j$ of $m \times m$ dimensions and auto-bias circuits NLCi of $n \times n$ dimensions; $\mathbf{U} = (U_1, \dots, U_m)^T$, $\Phi = (\varphi_1, \dots, \varphi_m)^T$ and $\tilde{\mathbf{E}} = (\tilde{E}_1, \dots, \tilde{E}_n)^T$ – vectors of oscillation amplitudes and phases of m dimension and vector of auto-bias voltage of n dimension;

$\mathbf{G}_R = \text{Re } \dot{\mathbf{G}} = |\dot{\mathbf{G}}| \cos \Delta\varphi^T$, $\mathbf{G}_I = \text{Im } \dot{\mathbf{G}} = |\dot{\mathbf{G}}| \sin \Delta\varphi^T$ – matrixes of real and imaginary units of equivalent complex conductance of active part of MPOS $|\dot{\mathbf{G}}| = (\bar{S}_{ji} K_{ji})_{j=1, i=1}^{m, n}$ of $m \times n$ dimensions;

$R = R_e \cdot K_\phi$ – matrix of reduced resistances;

$R_e = \text{diag}(R_{e_1}, \dots, R_{e_m})$ – matrix of equivalent resistances

$R_{e_j} = R_{in_\Sigma} \cdot R_{out_\Sigma} / (R_{in_\Sigma} + R_{out_\Sigma} + R_{q_j})$ of $m \times m$ dimensions, $\mathbf{K}_\phi = \text{diag}(K_{\phi_1}, \dots, K_{\phi_m})$ – matrix of equivalent phase link gains $K_{\phi_j} = 1 / \sqrt{1 + (\omega_{q_j} \cdot T_{\phi_j})^2}$,

$T_{\phi_j} \approx R_{in_\Sigma} (C_0 + C_{in_\Sigma}) + R_{out_\Sigma} (C_0 + C_{out_\Sigma})$ – time constant of equivalent phase link, providing phase shift $\Delta\varphi_{\phi_j} = -\text{arctg}(\omega_{q_j} \cdot T_{\phi_j})$ between immediate values of voltage $u(t)$ and current $i(t)$; ω_{q_j} , C_0 – resonance frequency and parallel capacity of QR;

$\Delta\varphi = (\Delta\varphi_{ji} + \Delta\varphi_{\phi_j})_{j=1, i=1}^{m, n}$ – matrix of $m \times n$ dimensions, which provides phase relations in excitation channels of MPOS;

$\Delta = \text{diag}(\Delta\omega_1 T_1, \dots, \Delta\omega_m T_m)$ – matrix of generalized detuning, $\Delta\omega_j = \omega_{q_j}^2 (R_{in_\Sigma} \tau_{in_\Sigma} + R_{out_\Sigma} \tau_{out_\Sigma}) / 2Q_{q_j} R_{q_j}$ – frequency adjustment to oscillation j explained by the fact that reduction was produced relative to natural frequencies of QR ω_{q_j} with no respect to phase-shift in FC circuits;

Q_{q_j} , R_{q_j} – quality factor and dynamic resistance of QR for oscillation frequency j ;

$R_{ab} = \text{diag}(R_{ab_1}, \dots, R_{ab_n})$ – matrix of auto-bias resistance of $n \times n$ dimensions;

$\mathbf{I}_0 = (I_{0_1}, \dots, I_{0_n})^T$ – vector of constant components of output currents $i_{out}(e_i)$;

$E_{(mm)}$, $E_{(m1)}$ – unit matrix of $m \times m$ dimensions and unit column vector $m \times 1$;

m – quantity of generating frequencies, n – quantity of excitation channels.

4. Dynamics of Multi-Frequency Piezoresonance Oscillation System

The case considers three-frequency excitation of the main mode of oscillations of multi-frequency QR and two additional ones – informative: temperature and vibration ($m=n=3$) [25, 26]. At polynomial approximation the output current of NLCi (fig. 1) can be represented as

$$i_{out_i}(e_i) = I_s \left[\hat{a}_0 + \sum_{r=1}^k \hat{a}_r (\hat{v}_i + X_{b_i})^r \right], \quad i = \overline{1, n}, \quad (13)$$

where $\hat{v}_i = v_i \cdot S_0 / I_s$ – normalized control voltage; $X_{b_i} = \bar{x}_{b_i} + \tilde{x}_{b_i}$, $\bar{x}_{b_i} = (E_i - E_s) \cdot S_0 / I_s$, $\tilde{x}_{b_i} = \tilde{E}_i \cdot S_0 / I_s$ – constant and variable components of normalized bias; $\hat{a}_0 = a_0 / I_s$, $\hat{a}_r = a_r \cdot I_s^{(r-1)} / S_0^r$ – normalized coefficients of approximating polynomial; I_s , E_s – the coordinates of the point in the center of current-voltage characteristic (CVC) with maximal gain slope S_0 ; $k = 3, 5$.

Having used trigonometry formulas of multiple argument and averaging accordantly to (6), the spectral components in NLCi input at approximation by polynomial of the third degree will be written as

$$I_{ji} = V_{ji} \cdot \bar{S}_{ji} = V_{ji} \cdot S_0 [A_{1i} + 0,75 \cdot \hat{a}_3 B_{1i}]; \quad (14)$$

$$I_{0_i} = I_s [A_{2i} + 0,5(\hat{a}_2 + 3\hat{a}_3 X_{b_i}) B_{2i}]. \quad (15)$$

Having substituted the expressions (14), (15) into truncated equations (10) – (12) and assuming $K_{\phi_j} \approx 1$ and $\Delta\varphi_{\phi_j} \approx 0$, the three-frequency MPOS ($m=3$) motion equation with three non-linear components ($n=3$) will be obtained:

$$\xi_j \frac{d\hat{U}_j}{dt} = S_0 R_{e_j} K_{jj} \cdot \hat{U}_j \times \left(\sum_{i=1}^3 (A_{1i} + 0,75 \hat{a}_3 B_{1i}) \cdot \gamma_{ji}^2 \frac{K_{ii}}{K_{jj}} - \frac{1}{S_0 R_{e_j} K_{jj}} \right); \quad (16)$$

$$\xi_j \frac{d\varphi_j}{dt} = S_0 R_{e_j} K_{jj} \cdot \sum_{i=1}^3 (A_{1i} + 0,75 \hat{a}_3 B_{1i}) \times \times \gamma_{ji} \sqrt{1 - \gamma_{ji}^2} \frac{K_{ii}}{K_{jj}} - \xi_j \Delta\omega_j; \quad (17)$$

$$\mu_i \frac{dX_{b_i}}{d\hat{t}} = -S_0 R_{ab_i} \times \left(A_{2i} + \frac{X_{b_i}}{S_0 R_{ab_i}} + 0,5(a_2 + 3a_3 X_{b_i}) B_{2i} \right) + \bar{x}_{b_i} . \quad (18)$$

The correspondent equations (16) – (18) at approximating transfer characteristics by polynomial of the fifth degree (13) are given in [16].

The following symbols used in expressions (14) – (18) stand for:

$$A_{1i} = \sum_{r=1}^3 r \hat{a}_r \cdot X_{b_i}^{(r-1)}, \quad A_{2i} = \hat{a}_0 + \sum_{r=1}^3 \hat{a}_r \cdot X_{b_i}^r,$$

$$B_{1i} = K_{ii}^2 \sum_{k=1}^3 b_{kij} \cdot \hat{U}_k^2, \quad B_{2i} = K_{ii}^2 \sum_{j=1}^3 \gamma_{ji}^2 \cdot \hat{U}_j^2;$$

$$b_{kij} = \gamma_{ki}^2, \text{ for } k = j \text{ and } b_{kij} = 2\gamma_{ki}^2, \text{ for } k \neq j;$$

$$\gamma_{ji} = \frac{K_{ji}}{K_{ii}} = \cos \Delta \varphi_{ji} - \text{coefficients of suppressing inter-channel interference (parasite) oscillations } (j \neq i);$$

$\hat{U}_j = S_0 U_j / I_s$ – normalized voltage in MQU output;

$\xi_j = T_j / \max_{1 \leq j \leq 3} T_j$, $\mu_i = T_{ab_i} / \max_{1 \leq j \leq 3} T_j$ – normalized time constants of partial oscillation system j and bias circuit i ;

$\hat{t} = t / \max_{1 \leq j \leq 3} T_j$ – normalized time; $j = \overline{1,3}$, $i = \overline{1,3}$.

Activity of excited oscillations is determined by regeneration parameter to be found from (16) when $\hat{U}_j = 0$:

$$K_j^{reg} = R_{e_j} K_{jj} S_{s_j} = R_{e_j} K_{jj} S_0 \times \sum_{i=1}^3 \left(\sum_{r=1}^{3,5} r \hat{a}_r X_{s_j}^{(r-1)} \right) \gamma_{ji} \frac{K_{ii}}{K_{jj}}, \quad (19)$$

where S_{s_j} – gain slope of volt-amps diagram in still point for oscillation j ; X_{s_j} – bias in the still point for NLC $_i$, being the solution for non-linear equation:

$$S_0 \cdot R_{ab_i} \left(\hat{a}_0 + \frac{X_{b_i}}{S_0 \cdot R_{ab_i}} + \sum_{r=1}^3 \hat{a}_r \cdot X_{b_i}^r \right) - \bar{x}_{b_i} = 0. \quad (20)$$

Figures 2 and 3 demonstrate distinguished case of stabilizing oscillations in three-frequency MPOS, as result of numerical integrating equations (16) – (18) in MATLAB system. The bipolar transistors (2SC9018) are used as active components (NLC $_i$), which have average statistical values of CVC parameters: $S_0 = 0.1$ A/B ; $\hat{a}_0 = 0.95$;

$\hat{a}_1 = 0.55$; $\hat{a}_2 = 0.051$; $\hat{a}_3 = -0.054$, when approximation error $\mathcal{E} \leq 5\%$. Initial values of low-varying parameters made: $T_j = 0.1$ sec; $T_{ab_i} = 1 \cdot 10^{-4}$ sec; $R_{ab_i} = 1 \text{ k}\Omega$; $K_{jj} = -20$ dB; $\bar{x}_{b_i} = 30$.

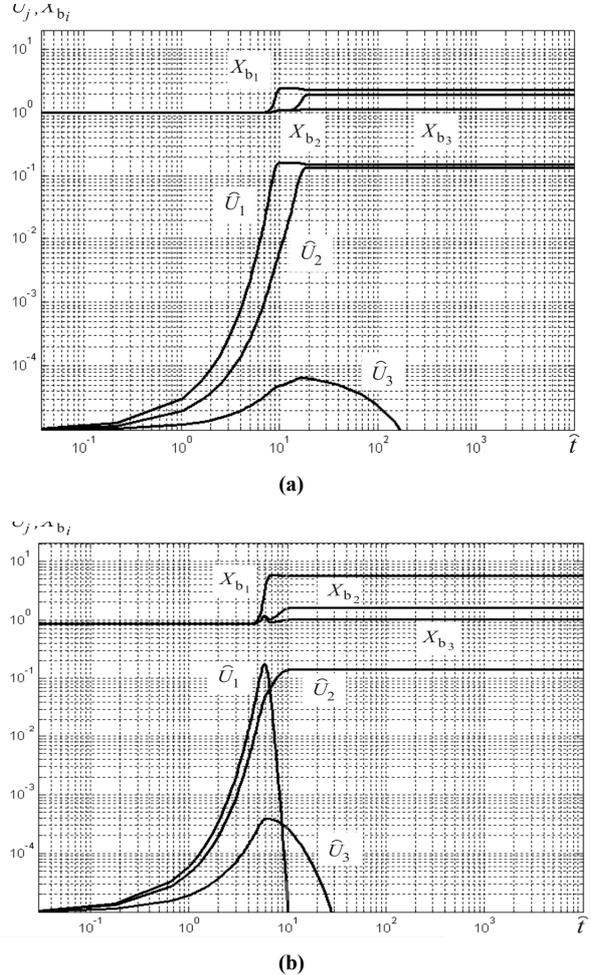
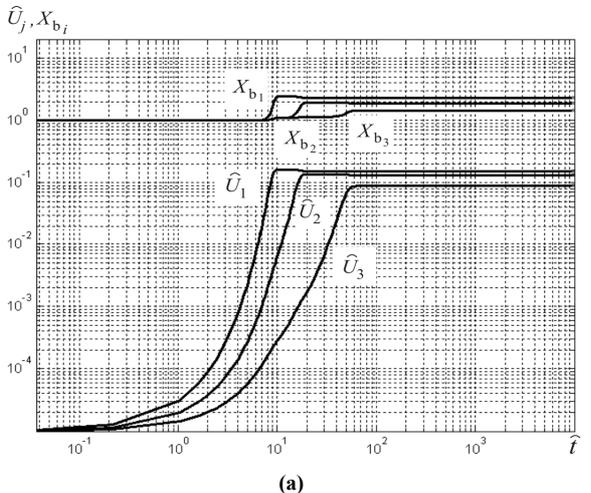


Figure 2. Distinguished case of appearing competition of oscillations in excitation channels of MPOS: suppressing energetically less active oscillations (a); suppressing energetically more active oscillation by less active one (b).



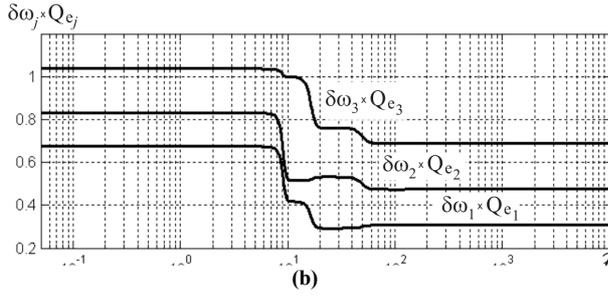


Figure 3. Stable three-frequency generation mode: setting oscillation amplitudes and auto-bias voltages (a); initial run-out of oscillation frequencies, caused by amplitude and phase conversion in excitation channels (b).

It's obvious that high competition in excitation channels ($\gamma_{ji} = -6\text{dB}$) can bring to situations when oscillations with low energetic activity (oscillation \hat{U}_3 , fig. 2, a, b) can be suppressed by more active oscillations (with high regeneration coefficient K_j^{reg}). At the same time the overly high increasing regeneration parameter K_j^{reg} can cause a breakdown of more energetically active oscillations (oscillation \hat{U}_1 , fig. 2, b) in excitation channels of MPOS. Reducing competition of oscillations due to increasing selective properties of FC circuits down to $\gamma_{ji} = -12\text{ dB}$ provides stable three-frequency mode of oscillation (fig. 3, a) even when regeneration coefficient values $K_1^{reg} = 1.25 K_2^{reg} = 1.5 K_3^{reg}$ are growing significantly [23].

In this case it assumes using the term of low or high active oscillation (oscillations) in relation to the other oscillations of MPOS, which, being energetically all-sufficient in one-frequency mode, may fade on transiting to multi-frequency mode of oscillations in the certain conditions.

Figure 4, a represents approximated relationships of oscillation stabilizing time $\tau_{stat} = \max_{1 \leq j \leq m} \tau_{stat_j}$ at various partial coefficients of FBC. It also has bifurcation curve that determines a range of stable MOM. Evidently, at high values of variations $\chi = \max_{1 \leq j \leq m} K_j^{reg} / \min_{1 \leq j \leq m} K_j^{reg} = \max_{1 \leq j \leq m} R_{e_j} / \min_{1 \leq j \leq m} R_{e_j}$ of regeneration coefficients the oscillation stabilizing time τ_{stat} is significantly increasing (by an order and more!). Compensation of this events when having the great scatter of equivalent resistances R_{e_j} is possible by selecting appropriate coefficients K_{jj} of partial FBC (increasing energy of oscillations), what it's not only improving dynamic properties of MPOS, but also brings to extending range of stable MOM (fig. 3, a).

The quite important characteristic of MPOS dynamics is group run-out of oscillation frequencies $\delta\omega_j Q_{e_j}$, which is defined according to (17). The modeling results indicate that amplitude-phase conversion slightly exhibits for energeti-

cally “weak” oscillations (fig. 3, b). At joint control of oscillation amplitude and voltage of auto-bias the relative run-out of frequencies $\delta\omega_j = (\omega_{q_j})^{-1} \cdot \Delta\omega_j(\hat{t}) = (\omega_{q_j})^{-1} \cdot \frac{d\varphi_j}{d\hat{t}}$ may reach values from 10^{-8} to 10^{-6} , and significantly reduce on increasing coefficients K_{jj} of partial circuits of FC (fig. 4, b).

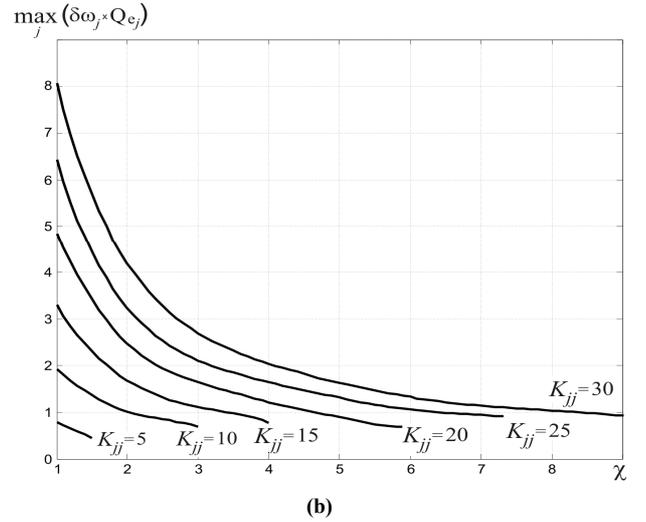
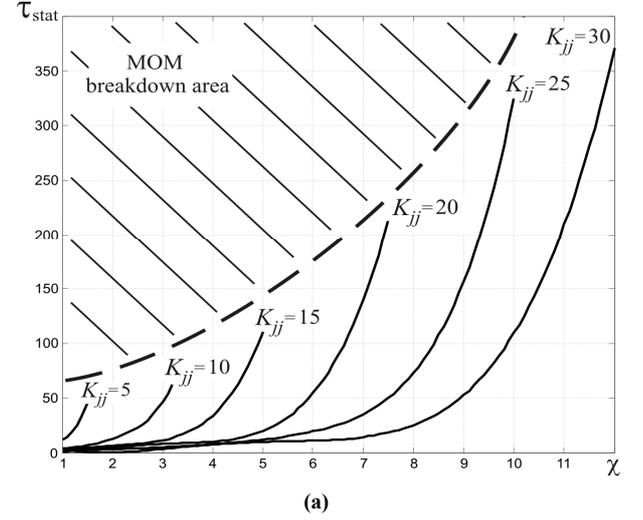


Figure 4. Dependences of normalized oscillation stabilizing time (a) and initial frequency run-out on MPOS parameters (b).

5. Conclusion

The paper proposes the mathematical model of multi-frequency piezoresonance oscillation system – PRD core, which enables to broaden the functional possibilities of multi-frequency – algorithmic methods to provide the invariance of piezoresonance devices. On basis of MPOS structure (fig. 1), which is realized on principles of building filter schemes, the generic truncated amplitude and phase equations (10)-(12) of system movements are derived. The use of generic equations allows removing restrictions on the quantity of generated frequencies while analyzing the

MPOS dynamics.

On the example of three-frequency oscillation mode the numerical analysis of equations of MPOS movement is done, and the margins of acceptable system parameters are defined, which provide the stable multi-frequency oscillation mode. The character cases of competitiveness of oscillations in excitation channels are presented (fig. 2,3). It is demonstrated that because of high competitiveness of oscillations not only suppression of energetically less active oscillations, but under certain conditions the suppression of energetically more active oscillation by less active one is possible. As the same time, increasing selective properties of FBC chains provides constant three-frequency oscillation mode even under considerably different meanings of regeneration coefficients ($K_1^{reg} = 1.25 K_2^{reg} = 1.5 K_3^{reg}$). This enables not only to manage each separate oscillation mode of quartz resonator, but also change the quantity of simultaneously generated frequencies.

Numerical analysis of equation of movement MPOS demonstrated that under large dispersions of coefficients of regeneration the group time of oscillation determination increases substantially (by order and more!). Compensation of these effects under large dispersion of equivalent resistances is possible by choosing appropriate coefficients of partial FC (by increasing the oscillation power), which not only improves the dynamic properties of MPOS, but also leads to widening the margins of stable MOM (fig. 4).

Rather important characteristic of MPOS is group frequency overshoot $\delta\omega_j = \left(\omega_{q_j}\right)^{-1} \cdot \Delta\omega_j(t)$. The results of modeling show that amplitude-phase conversion develops little for energetically less active oscillations. Besides minimization of time of MPOS developing oscillation can be provided by means of balancing activeness of generated oscillations (symmetric mode) under the condition of compromise between amplitudes of oscillations (sustainability) and initial frequency overshoot (stability).

Received results demonstrate that the use of developed mathematical model of MPOS, which contains inherently truncated differential equations for amplitude, phases and auto-bias voltageas predictive standard model of PRD core, allows to form a new approach to building piezoresonance units with controlled dynamics [27-30], represented in the form of adaptive controlled systems with predictive standard model and develop on their basis a new class of invariant to destabilizing disturbing PRD factors. Inherently such approach has the principle of using natural duplication in multi-frequency basis of PRD core – multi-frequency piezoresonance oscillation system, which enables not only to synthesize the system with current identification of disturbing factors on basis of theory of invariance, but also carry out the adaptation of PRD in accordance with their effects.

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